II A

①

Non-Abelian Gauge Theories.
(Yang-Mills).

U(1) gauge invce.

$$\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}.$$

$$D_\mu \psi = (\partial_\mu - ieA_\mu)\psi.$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

Invariant under.

$$A_\mu(x) \to A_\mu(x) + e^{-1}\partial_\mu \alpha(x) \quad , \quad \psi(x) \to \psi'(x) = e^{i\alpha(x)}\psi(x).$$

Note covariant derivative

transforms as

$$D_\mu\psi(x) \to (D_\mu\psi)'(x) = e^{+i\alpha(x)} D_\mu\psi(x).$$

i.e covariantly.

Abelian gauge theory U(1) gp.

Note $D_\mu D_\nu - D_\nu D_\mu = -ie\, F_{\mu\nu}$.

Generalize to non-Abelian gp. G.
(compact matrix)

$$\psi(x) \to \psi'(x) = U(\underline{\theta})\psi(x).$$

$$= e^{i\underline{\theta}\cdot\underline{T}}\,\psi(x).$$

If $G = SU(n)$ ~~we~~ and we put ψ in the defining rep $\psi(x) = \begin{pmatrix}\psi_1 \\ \vdots \\ \psi_n\end{pmatrix}$

$\underline{T}$ are $n \times n$ traceless matrices.

With $$[T_i, T_j] = i\, c_{ij}{}^{k}\, T_k.$$

↑ structure consts.

Normalize $\mathrm{tr}(T_i T_j) = \frac{1}{2}\delta_{ij}$ in fundamental.

Construct Lagrangian invariant under local $SU(n)$.

i.e $\underline{\theta}(x)$ functions of space-time

Need to introduce gaugefield corresponding to each parameter.

i.e A_μ^i $\quad i=1,\ldots,\dim$ of $\mathcal{G}$.

thus in $SU(n)$ $\quad i=1,\ldots,n^2-1$.

Introduce the matrix valued field

$$\underline{A}_\mu = \sum_{i=1}^{n^2-1} T_i A^i_\mu(x)$$

Define covariant derivative as in Abellian case, $\quad D_\mu\psi(x) = (\underline{\mathbb{1}}\partial_\mu - ig\underline{A})\psi(x)$

Require covariance i.e under $\psi \to \mathcal{U}(\theta)\psi$

$$D_\mu\psi(x) \to (D_\mu\psi)'(x) = \mathcal{U}(\theta)D_\mu\psi(x).$$

i.e $\quad (\partial_\mu - igA'(x))\psi'(x) = \mathcal{U}(\theta)(\partial_\mu - igA_\mu(x))\psi(x)$

or $\quad \mathcal{U}^{-1}(\partial_\mu - igA'(x))\mathcal{U}(\theta)\psi = (\partial_\mu - igA(x))\psi(x)$

i.e $\quad \mathcal{U}^{-1}\partial_\mu\mathcal{U}\psi + \partial_\mu\psi - ig\mathcal{U}^{-1}A'_\mu\mathcal{U}\psi = \partial_\mu\psi - igA_\mu\psi$

or $\quad ig\mathcal{U}^{-1}A'_\mu\mathcal{U} = igA_\mu + \mathcal{U}^{-1}\partial_\mu\mathcal{U}.$

i.e $\quad A_\mu(x) \to A'_\mu(x) = \mathcal{U}A_\mu\mathcal{U}^{-1} - \frac{i}{g}\partial_\mu\mathcal{U}\mathcal{U}^{-1}.$

Note this reduces to transf/ⁿ on p. ① for Abelian case

Infinitesimal transformations.

$$\mathcal{U}(\theta) = 1 + i\,\underline{\theta}(x)\,\underline{T} \qquad \mathcal{U}^{-1} = 1 - i\,\underline{\theta}(x)\,\underline{T}.$$

$$-\frac{i}{g}\,\partial_\mu \mathcal{U}\,\mathcal{U}^{-1} = \frac{1}{g}\,\partial_\mu \theta(x).\underline{T}$$

$$\mathcal{U}\,\underline{A}_\mu\,\mathcal{U}^{-1} = (1 + i\,\underline{\theta}.\underline{T})\,\underline{A}_\mu\,(1 - i\,\underline{\theta}.\underline{T}).$$

$$= \underline{A}_\mu + i\,[\underline{\theta}.\underline{T}, \underline{A}_\mu]$$

So $\underline{A}_\mu(x) \longrightarrow \underline{A}'_\mu(x) = \underline{A}_\mu(x) + i\,[\underline{\theta}.\underline{T}, \underline{A}_\mu] - \frac{i}{g}\,\partial_\mu \underline{\theta}(x).\underline{T}$

or $A^i_\mu \to A'^i_\mu = A^i_\mu - c^i{}_{jk}\,\theta^j A^k_\mu + \frac{1}{g}\,\partial_\mu \theta^i.$

i.e (for const. θ) A^i_μ transforms in the adjoint rep.

To define the field strength $\underline{F}_{\mu\nu}$ we again take the covariant derivative commutator.

$$[\underline{D}_\mu, \underline{D}_\nu]\psi = [\partial_\mu - ig\underline{A}_\mu, \partial_\nu - ig\underline{A}_\nu]\psi$$

$$= \{[\partial_\mu,\partial_\nu] - ig[\underline{A}_\mu,\partial_\nu] - ig[\partial_\mu,\underline{A}_\nu] - g^2[\underline{A}_\mu,\underline{A}_\nu]\}\psi$$

$$\equiv -ig\,\underline{F}_{\mu\nu}\psi$$

or $$\underline{F}_{\mu\nu} = \partial_\mu\underline{A}_\nu - \partial_\nu\underline{A}_\mu - ig[\underline{A}_\mu,\underline{A}_\nu]$$

or $$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\,c^{ijk}A^j_\mu A^k_\nu$$

Since $D_\mu\psi \to (D^\theta_\mu\psi)' = \mathcal{U}(\theta)D_\mu\psi$.

$$([D_\mu,D_\nu]\psi)' = \mathcal{U}(\theta)[D_\mu,D_\nu]\psi = -ig\,\mathcal{U}(\theta)\underline{F}_{\mu\nu}\psi$$

$$= [D_\mu\ D_\nu]'\,\mathcal{U}(\theta)\psi = -ig\,\underline{F}'_{\mu\nu}\,\mathcal{U}(\theta)\psi$$

ψ arbitrary for so $$\underline{F}'_{\mu\nu}(x) = \mathcal{U}(\theta)\,\underline{F}_{\mu\nu}(x)\,\mathcal{U}^{-1}(\theta).$$

Or for an infinitesimal transf^n.

$$\underline{F}'_{\mu\nu} = \underline{F}_{\mu\nu} + i[\underline{\theta}(x)\cdot T, \underline{F}_{\mu\nu}]$$

ie $$F'^i_{\mu\nu}T^i = F^i_{\mu\nu}T^i + i\theta^i\left[\frac{T^i}{\cancel{2}}, \frac{T^j}{\cancel{2}}\right]F^j_{\mu\nu}$$

⑦

or $F'^{i}_{\mu\nu} = F^{i}_{\mu\nu} - \theta^{j} C^{ijk} F^{k}_{\mu\nu}$

i.e. $\underline{F}_{\mu\nu}$ (like $\underline{A}$ for const θ) transform in adjoint representation (the transformation matrices representing the algebra are the C^{ijk}'s. $(T^{i})^{jk} \to C^{ijk}$.

From the transformation law the only invariant kinetic term (quadratic in derivatives) that we can form is:

$$\mathrm{tr}\, F_{\mu\nu}F^{\mu\nu} = \frac{1}{2} F^{i}_{\mu\nu} F^{i\mu\nu}$$

if T_i in fundamental

$\to \frac{1}{C(r)}$ if rep $\underline{r}$

* One cannot write down a free Lagrangian which is gauge invariant for a non-abelian gauge theory.

The Yang-Mills "photon" is charged unlike the abelian photon (the real one). It belongs to the adjoint rep of Group and local gauge invariance requires the Lag be constructed out of $F_{\mu\nu}$ not A_μ

* In particular no invariant mass term (or more generally not 1) for A_μ.

Pure Y-M theory

$$S = \int d^4x \left(-\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} \right)$$

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g C^{ijk} A^j_\mu A^k_\nu$$

This has trinear and quadrilear couplings.

$$\left(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g C^{ijk} A^j_\mu A^k_\nu\right)\left(\partial^\mu A^{i\nu} - \partial^\nu A^{i\mu} + g C^{ijk} A^{j\mu} A^{k\nu}\right)$$

$$= \left(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu\right)\left(\partial^\mu A^{i\nu} - \partial^\nu A^{i\mu}\right)$$

free K. E term
Lagrangian

Trilinear $$+ 2g\left(\partial_\mu A^i_\nu - \partial_\nu A^i_\mu\right) A^{j\mu} A^{k\nu} C^{ijk}$$

quartic $$+ g^2 C^{ijk} C^{ilm} A^j_\mu A^k_\nu A^{l\mu} A^{m\nu}$$

These couplings exist because of non-linear terms in F which in turn arise from fact that A^i carries SU(N) charge i.e. transforming non-trivially (in adjoint rep) of SU(N).

Yang-Mills coupled to matter.

$$\mathcal{L} = -\frac{1}{4}F^{i}_{\mu\nu}F^{i\mu\nu} + \bar{\Psi}\, i\gamma_\mu(\partial^\mu \underline{1} - ig\underline{A}^{\mu})\Psi$$

$$+ m\bar{\Psi}\Psi$$

$$= -\frac{1}{4}F^{i}_{\mu\nu}F^{i\mu\nu} + \bar{\Psi}(i\underline{\not{D}} - m)\Psi$$

** Note Independent coupling for each simple (also) factor of gauge group!

$\bar{\Psi}$ ~~matter coupling term~~

$$D_\mu = \partial_\mu - ig\underline{A}_\mu$$

$$-ig\bar{\Psi}\, i\gamma^\mu A^i_\mu T^i \Psi$$

$$= +g A^i_\mu \bar{\Psi}\gamma^\mu T^i\Psi$$

Note that unlike the case of abelian gauge theories the coupling constant to matter is fixed by gauge invariance. The self interaction strength is the ~~same~~ and the matter coupling governed by same constant. Cannot couple a field with strength λg and preserve gauge invariance.

This is because in

$$[D_\mu, D_\nu] = -ig F_{\mu\nu}$$

g occurs in F ~~both sides~~. ⇒ non-linear relation for g.

For a <u>simple</u> group there can be only one coupling constant. For direct product group such as U(1) × SU(2) × SU(3) each simple factor has its own coupling const. g_1, g_2, g_3 No reason for equality.

Grand unification puts ~~g~~

$$U(1) \times SU(2) \times SU(3) \subset \underset{\uparrow}{SU(5)}$$

Simple group
one ~~g~~ coupling.
greater predictive power.

Note one can scale A to absorb g.

$$A \to g^{-1} A$$

$$D_\mu = \partial_\mu - i \underline{A}_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - g[\underline{A}_\mu, A_\mu].$$

$$\to \frac{1}{g} [\partial_\mu A_\nu - \partial_\nu A_\mu - [A_\mu, A_\nu]$$

$$= \frac{1}{g} F$$

scaling $\psi \to \frac{1}{g} \psi$

We may write

$$\mathcal{L} \to \frac{1}{g^2} \mathcal{L}\,(g=1).$$

In path integral

$$e^{i/\hbar\, S[A,\psi,g]}$$

$$\to e^{i/\hbar g^2\, S[A,\psi]}$$

↑ no g dependence

ie g^2 occurs same way that $\hbar$ does. - recall loop exp^n.

Equations of Motion

$$\mathcal{L} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \bar{\psi} i \gamma^\mu D_\mu \psi - m \bar{\psi}\psi .$$

$$D_\mu \equiv \partial_\mu - i g \underline{A}_\mu$$

$$\delta S = \int d^4x \Big[-\frac{1}{2} F^i_{\mu\nu} \big[\partial_\mu \delta A^i_\nu - \partial_\nu \delta A^i_\mu + g C^i_{jk} (A^j_\mu \delta A^k_\nu + \delta A^j_\mu A^k_\nu) \big]$$

$$+ g \bar{\psi} \gamma^\mu T^i \psi \, \delta A^i_\mu \Big] .$$

$$= \int d^4x \left[\partial^\mu F^i_{\mu\nu} - C^i{}_{jk} A^{j\mu} F^k_{\mu\nu} + g \bar{\psi} \gamma_\mu T^i \psi \right] \delta A^{i\nu} .$$

EoM $$\frac{\delta S}{\delta A^{i\nu}} = D^\mu F^i_{\mu\nu} + g \bar{\psi} \gamma_\nu T^i \psi = 0 .$$

$$\text{or} \quad D^\mu F^i_{\mu\nu} = -g \bar{\psi} \gamma_\nu T^i \psi \equiv -g j^i_\nu$$

$$\uparrow$$

analog of electric current.

$$D^\mu F^i_{\mu\nu} \equiv (\partial^\mu \delta^{ik} - C^{ijk} A^{j\mu}) F^k_{\mu\nu} .$$

$$\left[D^\nu D^\mu F_{\mu\nu} \right]^i = -g \left(D^\nu j_\nu \right)^i$$

LHS zero since

$$\left(D^\nu D^\mu F_{\mu\nu} \right)^i = \frac{1}{2} \left([D^\nu, D^\mu] F_{\mu\nu} \right)^i$$

$$= \frac{1}{2} C^{jik} F^j_{\mu\nu} F^k_{\mu'\nu'} \eta^{\mu\mu'} \eta^{\nu\nu'} = 0$$

From anti-symmetry of C^{ijk}.

Hence we have the <u>covariant</u> conservation law.

$$D^{\mu} j_{\mu}^{i} = 0 .$$

rather than $\partial^{\mu} j_{\mu} = 0$ as in QED.

Reason is that A_{μ}^{i} also carries charge.

Fermion eq^{n}. $\frac{\delta S}{\delta \bar{\psi}} = 0$.

$$(i\not{D} - m)\underline{\psi} = \{ i\gamma^{\mu} (\partial_{\mu} - ig \underline{A}_{\mu}) - m \} \underline{\psi} = 0.$$

Gauge covariant Dirac eq^{n} as in QED.

II A' Higgs Mechanism in SU(2) gauge Theory.

Fields Gauge: $W_\mu^r \to \underline{W}_\mu \equiv \sum_{r=1}^{3} W_\mu^r t^r$ $\quad t \equiv \frac{\sigma^r}{2}$

$\mathrm{tr}\, t^r t^s = \frac{1}{2}\delta^{rs}$, $\quad [t^r, t^s] = i\epsilon^{rsu} t^u$

Higgs field. $\underline{H} = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$ - SU(2) doublet.
H^+, H^0 - complex!

Covariant derivative $D_\mu H = \left(\partial_\mu 1 - ig\underline{W}_\mu\right) H.$

Field Strength $F_{\mu\nu}^r = \partial_\mu W_\nu^r - \partial_\nu W_\mu^r + g\epsilon^{rsu} W_\mu^s W_\nu^u$

Potential $V(H) = \cancel{-\mu^2 H^\dagger H} + \lambda\left(H^\dagger H - \frac{\mu^2}{2\lambda}\right)^2$

Note potential minimum at $\langle H^\dagger H \rangle = \frac{v^2}{2}$

$v \equiv \sqrt{\mu^2/\lambda}.$

SU(2) gauge Invariant

Lagrangian:

$$\mathcal{L} = (D_\mu H)^\dagger (D_\mu H) - V(H) - \frac{1}{4} F_{\mu\nu}^r F^{r\mu\nu}$$

Gauge transformations $\underline{H} \to \underline{H}' = e^{i t^r \theta^r(x)} \underline{H}.$

$u \equiv e^{i t^r \theta^r(x)}$ $\quad$ ~~A~~ $\underline{W}_\mu \to \underline{W}_\mu' = u \underline{W}_\mu u^{-1} - \frac{i}{g} \partial_\mu u\, u^{-1}.$

By an SU(2) transformation take

$$\langle H \rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\ v\end{pmatrix}$$

To display the spectrum of the theory (to zeroth order in $\hbar$).

Put $\underline{H}(x) = e^{i t^r \frac{\xi^r(x)}{v}} \begin{pmatrix}0\\ \frac{v+\eta}{\sqrt{2}}\end{pmatrix}$ ξ^r, η real.

with $\langle \xi \rangle = \langle \eta \rangle = 0$.   2 complex → 4 real

and $\underline{Y}_\mu \equiv U(\xi)\,\underline{A}_\mu\, U^{-1}(\xi) - \frac{i}{g}\partial_\mu U(\xi) U^{-1}(\xi)$

$$U(\xi) = e^{-i t^r \frac{\xi^r(x)}{v}}$$

This is completely equivalent to doing a gauge transformation with gauge parameter $\theta^r(x) = -\xi^r/v$!

i.e $\underline{H} \to \underline{H}' = \underline{U}(\xi)\underline{H} = \begin{pmatrix}0\\ \frac{v+\eta}{\sqrt{2}}\end{pmatrix} \equiv \underline{h}$.

$$W_\mu \to W_\mu' = \underline{U}\,\underline{W}_\mu \underline{U}^{-1} - \frac{i}{g}\partial_\mu \underline{U}\,\underline{U}^{-1} = Y_\mu .$$

$h^\dagger h = \frac{(v+\eta)^2}{2} = \frac{v^2}{2} + v\eta + \frac{\eta^2}{2}$

$h = \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$

So the Lagrangian can be written as

$$\mathcal{L} = \left(D_\mu \underline{h}\right)^\dagger \left(D^\mu h\right) - \lambda\left(h^\dagger h - \frac{\mu^2}{2\lambda}\right)^2 - \frac{1}{4} F^r_{\mu\nu} F^{r\mu\nu}$$

Redefined

$$F^r_{\mu\nu} = \partial_\mu Y^r_\nu - \partial_\nu Y^r_\mu + g\,\epsilon^{rsu} Y^s_\mu Y^u_\nu .$$

$V = -\lambda\eta^2\left(v + \frac{\eta}{2}\right)^2 = -\lambda^2 v^2 \eta^2 - \cancel{2}\lambda v \eta^3 -$

As in $U(1)$ Higgs mechanism the "Goldstone fields" ξ^r have disappered from $\mathcal{L}$. Note also.

$$D_\mu h = \left(\partial_\mu - igY^r_\mu t^r\right)\begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$$

And $\left(D_\mu h\right)^\dagger = \left(0, \frac{v+\eta}{\sqrt{2}}\right)\left(\overleftarrow{\partial}_\mu + igY^r_\mu t^r\right)$.

So $\left(D_\mu h\right)^\dagger\left(D_\mu h\right)$ has a mass term for Y_μ

i.e. $\cancel{\frac{1}{2}(gv)^2}\; g^2\left(0, \frac{v}{\sqrt{2}}\right)\frac{\sigma^r}{2}\frac{\sigma^s}{2}\begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} Y^r_\mu Y^{s\mu}$

$\sigma^r\sigma^s = \delta^{rs} + i\epsilon^{rsu}\sigma^s$

$$= \frac{1}{2}\left(\frac{gv}{2}\right)^2 Y^r_\mu Y^{r\mu}$$

$v^2 = \frac{\mu^2}{\lambda}$

corresponding to a mass $M_W = gv/2$.

Also $m_H = +2\lambda^2 v^2 = 2\lambda\mu^2$. (Notation write $M_Y \to M_W$, $M_\eta \to M_H$)

physical Higgs mass

II B

Constructing the Standard Model.

Useful to write the fermion fields as Left Chiral Dirac fields

$$\psi_D \equiv (\qquad \psi_{D\cdot L} = P_L \begin{pmatrix} \psi_L \\ \sigma \chi_L^* \end{pmatrix} = \begin{pmatrix} \psi_L \\ 0 \end{pmatrix}$$

and their charge conjugates

$$(\psi_D{}^c)_L = \begin{pmatrix} \chi_L \\ 0 \end{pmatrix}.$$

Note that these two are independent fields.

The local gauge symmetry of the standard model is

$$G = SU(3) \times SU(2)_L \times U(1)_Y.$$

$SU(2)_L$ ⇒ acts on left chiral fields.

$U(1)_Y$. – Abelian Symmetry - Weak Y-hypercharge

→ Simplest possible group consistent with the available experimental data and theoretical constraints circa 1970.

②

SM Lagrangian - 4 - parts.

$$\mathcal{L} = \underbrace{\mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{gauge}}}_{\text{Strong/Weak/EM.}} + \underset{\substack{\uparrow \\ \text{Higgs/Matter interactions.}}}{\mathcal{L}_{\text{Yukawa}}} + \underset{\substack{\uparrow \\ \text{Scalar} \\ \text{~~Higgs~~} \\ \text{pseudoscalar}}}{\mathcal{L}_{\text{Higgs}}}$$

Each separately Invariant under $\mathcal{G}$ (local)

$A = 1, 2, 3$

	Matter			SU(3).	$SU(2)_L$	$U(1)_Y$
$SU(2)_L$ doublets! $SU(3)$ triplets	$Q_L^A = \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$SU(2)_L$ singlet $SU(3)$ - Triplet	$(u^c)_L^A = (u^c)_L$	$(c^c)_L$	$(t^c)_L$	$\overline{3}$	1	$-\frac{2}{3}$
"	$(d^c)_L^A = (d^c)_L$	$(s^c)_L$	$(b^c)_L$	$\overline{3}$	1	$+\frac{1}{3}$.
All fields ~~$u_L = P_L$~~ Left-handed Dirac spinors.	$L_L^A = \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$.
	$(e^c)_L^A = (e^c)_L$	$(\mu^c)_L$	$(\tau^c)_L$	1	1	1.

Note

$Q = \frac{\sigma_3}{2} P_- + Y$

$$\mathcal{L}_{\text{Matter}} = i \bar{Q}_L^A \not{D} Q_L^A + i \overline{(u^c)}_L^A \not{D} (u^c)_L^A + i \overline{(d^c)}_L^A \not{D} (d^c)_L^A$$
$$+ i \bar{L}_L^A \not{D} L_L^A + i \overline{(e^c)}_L^A \not{D} (e^c)_L^A.$$

Not^n

$(\underline{n}, \underline{m})_Y$
$\uparrow \quad \uparrow$
$SU(3), SU(2)_L$

$$\not{D} \equiv \gamma^\mu D_\mu$$

$$D_\mu Q_L^A = \left(\partial_\mu - i g_3 \frac{\lambda^\alpha}{2} G_\mu^\alpha - i g \frac{\sigma^r}{2} W_\mu^r - \frac{i g'}{6} B_\mu\right) Q$$

$$D_\mu (u^c)_L^A = \left(\partial_\mu - i g_3 \left(\frac{-\lambda^{\alpha *}}{2}\right) G_\mu^\alpha - i g' \left(-\frac{2}{3}\right) B_\mu\right) (u^c)_L^A$$

etc.

$T^\alpha = \frac{\lambda^\alpha}{2}$
$t^r = \sigma^r/2$

In general

$$D_\mu \psi = \left(\partial_\mu - i g_3 T^\alpha G_\mu^A - i g t^r W_\mu^r - i g' Y B_\mu\right) \psi.$$

generator in
with T^α - $SU(3)$ rep. of ψ

t^r - $SU(2)$ rep of ψ.

Y - Hypercharge of ψ.

i.e the field ψ transforms under gauge transformations as.

$$SU(3) \quad \psi \to e^{i\theta^\alpha(x) T^\alpha}\psi \,, \quad SU(2): \psi \to e^{i\epsilon^r(x) t^r}\psi.$$

$$U(1)_Y : \quad \psi \to e^{i\varphi(x) Y}\psi$$

$$u = e^{i\theta^\alpha T^\alpha} \quad SU(3): \underline{G}_\mu \equiv G^\alpha_\mu \underline{T}^\alpha \to \underline{G}_\mu' = u G_\mu u^{-1} - \frac{i}{g_3}\partial_\mu u u^{-1}$$

$$u = e^{i\epsilon^r t^r} \quad SU(2)_L: \underline{W}_\mu \equiv W^\alpha_\mu \underline{t}^\alpha \to \underline{W}_\mu' = u W_\mu u^{-1} - \frac{i}{g}\partial_\mu u u^{-1}$$

$$B_\mu \to B_\mu' = B_\mu + \frac{1}{g'}\partial_\mu\varphi(x).$$

Using $(\psi^c)_L = C\gamma^0\psi^*_R$

we may rewrite $\mathcal{L}$ matter as.

check!

$$\mathcal{L}_{matter} = i\overline{Q}^A_L \not{D} Q^A_L + i\overline{u}^A_R \not{D} u^A_R + i\overline{d}^A_R \not{D} d^A_R + i\overline{L}^A \not{D} L^A + i\overline{e}^i_R \not{D} e^i_R$$

Note that there <u>cannot</u> be any

mass terms for the fermions

- No Majorana mass - All ψ's carry $Y \neq 0$

(Also in complex reps of $SU(3)$ and/or pseudo real $SU(2)$).

No Dirac mass term since that would require pairs of (say Left chiral) fermions in complex conjugate reps - and we don't have any.

(Recall. $\Delta \mathcal{L} = - m \bar{\psi} \psi = m \left(\xi_L^T \epsilon \psi_L - \psi_L^\dagger \epsilon \xi_L^* \right)$

for $\psi = \begin{pmatrix} \psi_L \\ \epsilon \xi_L^* \end{pmatrix}$)

Weak / E.M interactions.

$P_\pm = \frac{1 \pm \gamma_5}{2}$

$\psi = \begin{pmatrix} u \\ d \end{pmatrix}$

or $\begin{pmatrix} \nu \\ e^- \end{pmatrix}$

$\frac{\sigma^+}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$\frac{\sigma^-}{2} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$

$$\mathcal{L}_{INT} = \bar{\psi} \gamma^\mu \left(g \frac{\sigma^r}{2} W_\mu{}^r P_- + g' Y B_\mu \right) \psi$$

$$= \bar{\psi} \gamma^\mu \left(\frac{g}{\sqrt{2}} \frac{\sigma^+}{2} P_- W_\mu^+ + \frac{g}{\sqrt{2}} \frac{\sigma^-}{2} P_- W_\mu^- + g \frac{\sigma^3}{2} P_- W_\mu^3 \right.$$

$W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp i W^2)$

$$\left. + g' Y B_\mu \right) \psi .$$

$\sigma^\pm = \sigma^1 \pm i \sigma^2$

Write $\quad Y = Q - \frac{\sigma^3}{2} P_-$

check

$$Q P_- = \begin{pmatrix} 1/6 + 1/2 \\ 1/6 - 1/2 \end{pmatrix} = \begin{pmatrix} 2/3 \\ -1/3 \end{pmatrix}$$

$$\text{or } \begin{pmatrix} -1/2 + 1/2 \\ -1/2 - 1/2 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}.$$

$$Q P_+ = Y P_+ = \begin{pmatrix} 2/3 \\ 1/3 \end{pmatrix} P_+$$

$$\text{or } \begin{pmatrix} 1 \end{pmatrix}$$

Identify photon as field coupling to $Q = \frac{\sigma^3}{2} P_- + Y$.

Put $\quad W_\mu^3 = \sin\theta_W A_\mu + \cos\theta_W Z_\mu .$

$\quad B_\mu = \cos\theta_W A_\mu - \sin\theta_W Z_\mu .$

$$g \frac{\sigma^3}{2} P_- (\sin\theta_W A_\mu + \cos\theta_W Z_\mu) + g' Y (\cos\theta_W A_\mu - \sin\theta_W Z_\mu)$$

$$= A_\mu \underbrace{\left(g \sin\theta_W \frac{\sigma^3}{2} P_- + g' \cos\theta_W Y \right)}_{} + Z_\mu \left(g \cos\theta_W \frac{\sigma^3}{2} P_- - g' \sin\theta_W Y \right)$$

should be prop. to $Q = \frac{\sigma^3}{2} P_- + Y$.

$$\Rightarrow \quad \frac{g'}{g} = \tan\theta_W \quad \text{i.e} \quad \sin\theta_W = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$\cos\theta_W = \frac{g}{\sqrt{g^2 + g'^2}}$$

So the interaction (matrix) is

$$\frac{gg'}{\sqrt{g^2+g'^2}} A_\mu Q + \frac{1}{\sqrt{g^2+g'^2}} Z_\mu \left(g^2 \frac{\sigma^3}{2} P_- - g'^2 Y \right).$$

$$\Rightarrow \frac{gg'}{\sqrt{g^2+g'^2}} = e \quad \Rightarrow g = \frac{e}{\sin\theta_W} \quad g' = \frac{e}{\cos\theta_W}.$$

Alternative form of Interaction matrix.

$$e A_\mu Q + e Z_\mu \left(\cot\theta_W \frac{\sigma^3}{2} P_- - \tan\theta_W \, Y \right).$$

$$\overline{Q} \qquad Y = Q - \frac{\sigma^3}{2} P_-$$

$$\frac{\cos\theta_W}{\sin\theta} + \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$= e A_\mu Q + e Z_\mu \left((\cot\theta_W + \tan\theta_W) \frac{\sigma^3}{2} P_- - \tan\theta_W Q \right)$$

$$e = g \sin\theta_W$$

$$= e A_\mu Q + \frac{g}{\cos\theta_W} Z_\mu \left(\frac{\sigma^3}{2} P_- - \sin^2\theta_W Q \right).$$

$$\text{So } \mathcal{L}_{int} = e A_\mu J^\mu_{E.M.} + \frac{g}{\cos\theta_W} Z_\mu J^{0\mu}$$

Neutral wk current

$$J_\mu^0 = \bar{\psi} \gamma_\mu \left(\frac{\sigma^3}{2} P_- - \sin^2\theta_W Q \right) \psi.$$

$$J_{\mu E.M} = \bar{\psi} \gamma_\mu Q \cdot \psi \qquad \psi = \binom{\nu}{e} \text{ or } \binom{u}{d}. \text{ etc.}$$

Yukawa couplings. becomes.

$$\mathcal{L}_{Yukawa} = - f_u^{AB}\, \bar{Q}^A_{Li}\, \epsilon_{ij}\, H^*_j\, u^B_R - f_d^{AB}\, \overline{Q^A_{Li}}\, H_i\, d^B_R$$
$$- f_e^{AB}\, \bar{L}^A_{Li}\, H_i\, e^B_R \quad + h.c.$$

$e' \to e$

etc

$H \to \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}$

$$= -\frac{v}{\sqrt{2}} \left(f_u \bar{e}_L e_R + f_d \bar{d}_L d_R + f_e \bar{e}_L e_R \right) + h.c.$$
$$- \frac{\eta(x)}{\sqrt{2}} \left(f_u \bar{u}_L u_R + f_d \bar{d}_L d_R + f_e \bar{e}_L e_R \right) + h.c.$$

After diagonalizing the f^{AB} matrices – see next section.

eg: $$f^u \bar{Q}^u_L \epsilon H^* u_R = f^u (\bar{u}_L, \bar{d}_L) \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \frac{v+\eta}{\sqrt{2}} u_R$$
$$= \frac{1}{\sqrt{2}} f^u (v+\eta) \bar{u}_L u_R .$$

(Note $(\bar{u}_L u_R)^\dagger = \bar{u} \frac{(1-\gamma_5)}{2} u$.

$\bar{u}_L u_R = \bar{u} \frac{(1+\gamma_5)}{2} u$.

So if f^u real then ~~$\bar{u}\frac{v f}{\sqrt{2}}$~~ $\frac{1}{\sqrt{2}} f^u (v+\eta) \bar{u}_L u_R + h.c.$

$$= \frac{1}{\sqrt{2}} f^u (v+\eta) \bar{u} u .$$

$\Rightarrow m_u = f^u \frac{v}{\sqrt{2}}$

etc.

Last piece is the Higgs Sector

SU(3) - Singlet
SU(2) - doublet
Hypercharge $\frac{1}{2}$.
$H \in (1,2)_{1/2}$

$$V_{Higgs} = + \lambda \left(H^\dagger H - \frac{\mu^2}{2\lambda}\right)^2 \qquad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$Y_H =$

As in previous discussion (for SU(2) case).

$$H_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v^2 = \mu^2/\lambda$$ at the minimum of the potential.

Put

$$H(x) = U^{-1}(\xi) \begin{pmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{pmatrix}.$$

$$U(\xi) = e^{i(\xi^1 \frac{\sigma^1}{2} + \xi^2 \frac{\sigma^2}{2} + \xi^3 \bar{Q})} \qquad \vec{\xi}_0 = \eta_0 = 0$$

$\frac{\sigma^1}{2}$, $\frac{\sigma^2}{2}$ and $\bar{Q} = \cot\theta_W \frac{\sigma^3}{2} - \tan\theta_W Y$

are the 'broken' generators in $SU(2)_L \times U(1)_Y$

As argued before $U(\xi)$ dependence disappears due to Gauge invariance. So

$$\mathcal{L}_{Higgs} = (D_\mu H)^\dagger (D^\mu H) - V(H)$$

$v^2 = \frac{\mu^2}{\lambda}$.

with $D_\mu H = \left(\partial_\mu - ig\frac{\vec{\sigma}}{2}\vec{W}_\mu - i\frac{g}{2}B_\mu\right)\left(\frac{v+\eta}{\sqrt{2}}\right)\chi \qquad \chi \equiv \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$$V(H) = \lambda v^2 \eta^2 + \lambda v \eta^3 + \frac{\lambda \eta^4}{4} \Rightarrow m_\eta^2 = 2\lambda v^2$$

The Vector Boson Masses,

$\chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$\frac{\sigma^+}{2}\chi = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix}$
$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

$\chi^\dagger \frac{\sigma^-}{2} = (1, 0)$

$\frac{\sigma^-}{2}\chi = 0$

$W^{\pm}_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)$

$Q = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$Q\chi = 0$

can be read off from the $(D_\mu H)^\dagger D^\mu H$ term of $\mathcal{L}_{Higgs}$.

after putting again $H \to \frac{V}{\sqrt{2}}\chi$ $\quad \chi = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$\frac{V^2}{2}\chi^\dagger\left(g\frac{\vec{\sigma}}{2}\cdot\vec{W}_\mu - \frac{g'}{2}B_\mu\right)\left(g\frac{\vec{\sigma}}{2}\cdot\vec{W}^\mu - \frac{g'}{2}B_\mu\right)\chi.$$

$$= \frac{V^2}{2}\chi^\dagger\left(g\frac{\sigma^+}{2\sqrt{2}}W^+_\mu + g\frac{\sigma^-}{2\sqrt{2}}W^-_\mu + \frac{g}{\cos\theta_W}Z_\mu\left(\frac{\sigma^3}{2} - \sin^2\theta_W Q\right) + eA_\mu Q\right)$$

$$\times\left(g\frac{\sigma^+}{2\sqrt{2}}W^{\mu +} + \frac{g\sigma^-}{2\sqrt{2}}W^{\mu -} + \frac{g}{\cos\theta_W}Z^\mu\left(\frac{\sigma^3}{2} - \sin^2\theta_W Q\right) + eA^\mu Q\right)\chi.$$

<u>Note</u> $\quad \frac{\sigma^-}{2}\chi = 0 \quad Q\chi = 0 \quad \chi^\dagger Q = 0 \quad \frac{\sigma^+}{2}\chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\frac{\sigma^3}{2}\chi = -\frac{1}{2}\chi$$

So this reduces to.

$$\frac{g^2v^2}{4}W^+_\mu W^{-\mu} + \frac{g^2v^2}{4\cos^2\theta_W}\frac{1}{2}Z_\mu Z^\mu.$$

From $|D_\mu H|^2$ term identify. $\textcircled{4F}$.

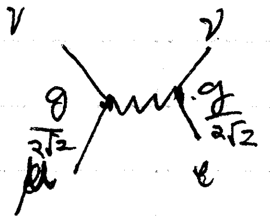
So $$M_W^2 = \frac{g^2 v^2}{4} \qquad M_Z^2 = \frac{g^2 v^2}{4\cos^2\theta_W}.$$

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1$$

(ρ - parameter) is a consequence of having just one Higgs doublet.

If we use 2 - doublets this would change.

Value of v from Fermi coupling.



$$\frac{G_F}{\sqrt{2}} \simeq \cancel{g}^2$$

From charged current interaction terms.

$$\mathcal{L}_{I\,C.C} = \frac{g}{\sqrt{2}}\left(J_\mu^+ W^{+\mu} + J_\mu^- W^{-\mu}\right) \qquad J_\mu^\pm = \bar{\Psi}\gamma^\mu \frac{\sigma^\pm}{2} P_- \Psi.$$

If only one generation.

$$\Psi = \begin{pmatrix} \nu_e \\ e \end{pmatrix} \text{ or } \begin{pmatrix} u \\ d \end{pmatrix} \qquad J_\mu^\pm = \frac{1}{2}\bar{\nu}\gamma_\mu(1-\gamma_5)e + \frac{1}{2}\bar{u}\gamma_\mu(1-\gamma_5)d$$

etc.

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2}{8 g^2 v^2/4} = \frac{1}{2v^2} \qquad G_F^{-1} = 10^5\,\mathrm{GeV}^2 \Rightarrow v = 250\,\mathrm{GeV}.$$

$$M_W^2 = \frac{e^2 v^2}{4\sin^2\theta_W} \simeq \left(\frac{37\,\text{GeV}}{\sin\theta_W}\right)^2$$

From $g = \frac{e}{\sin\theta_W}$ $\frac{e^2}{4\pi} \simeq \frac{1}{137}$.

and $$M_Z^2 = \frac{e^2 v^2}{4\sin^2\theta_W\cos^2\theta_W} = \left(\frac{37\,\text{GeV}}{\sin\theta_W\cos\theta_W}\right)^2$$

If we know $\sin\theta_W$ then M_Z, M_W are predictions

Alternatively if M_W (say) is known M_Z, $\sin\theta_W$ are predictions.

Neutral current Expts (these are also predictions!) determined $\sin^2\theta_W \simeq 0.22$. (CERN 1972 expt)

$\Rightarrow$ $M_W \simeq 80\,\text{GeV}$ $M_Z \simeq 90\,\text{GeV}$.

discovered CERN UA1 expt. 1979

Current values.

LEP $M_W = 80.398(25)\,\text{GeV}/c^2$ $\sin^2\theta_W = 0.23119(14)$.

$M_Z = 91.1876(21)\,\text{GeV}/c^2$ at Z-pole.

~~Note if we use above value for θ_W & $\alpha\left(\overset{(100\,\text{GeV})}{M_Z}\right) \simeq \frac{1}{128}$~~

Global symmetries:

Accidental symmetries of $\mathcal{L}_{matter}$. – follows from gauge invariance and rep. content. i.e not <u>imposed</u> as an additional symmetry.

$A, B = 1, 2, 3.$

$UU^\dagger = U^\dagger U = 1$

$$Q_L^A \to U_Q^{AB} Q_L^B \qquad u_R^A \to U_u^{AB} u_R^B$$

$$d_R^A \to U_d^{AB} d_R^B \qquad L_L^A \to U_L^{AB} L_L^B, \; e_R^A \to U_e^{AB} e_R^B$$

Globaly symm. of $\mathcal{L}_{matter}$

$$U(3)^5.$$

Violated (mostly) by $\mathcal{L}_{Yukawa}$

$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

$(1, 2)_{1/2}$

Under $SU(3) \times SU(2) \times U(1)$.

$$\mathcal{L}_{Yukawa} = -f_u^{AB} \bar{Q}_{Li}^A \epsilon_{ij} H_j^* u_R^B - f_d^{AB} \overline{Q_{Li}^A} H_i d_R^B - f_e^{AB} \overline{L_{Li}^A} H_i e_R^B$$

$$+ h.c.$$

Only Baryon number and Lepton # survive

B:

$$Q_L^A \to e^{i\theta/3} Q_L^A \qquad L_L^A \to e^{i\varphi} L_L^A$$

$$u_R^A \to e^{i\theta/3} u_R^A, \; d_R^A \to e^{i\theta/3} d_R^A \qquad e_R^A \to e^{i\varphi} e_R^A.$$

As in general discussion of Higgs phenomenon - spontaneous symmetry breakdown (SSB)

$$\Rightarrow \quad \langle H \rangle \equiv H_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}.$$

Fermions (except for ν) acquire mass terms.

$$\mathcal{L}_M = - m_u^{AB} \bar{u}_L^A u_R^B - m_d^{AB} \bar{d}_L^A d_R^B - m_e^{AB} \bar{e}_L^A e_R^B .$$

$$+ h.c.$$

$$m_\bullet^{AB} = f_\bullet^{AB} \frac{v}{\sqrt{2}} . \qquad \bullet = u, d\, e.$$

Yukawa matrices $f_\bullet^{AB}$ are 3- complex

3×3 - So $3 \times 3 \times 3 \times 2 = 54$

new real parameters!

<u>**But**</u>.

Many - Redundant. - removed by field redefinitions - Unitary transformations - preserve Kinetic terms!

$$(\bar{\psi}_L \not{\partial} \psi_L \rightarrow \bar{\psi}_L U^\dagger \not{\partial} U \psi_L = \bar{\psi}_L \not{\partial} \psi_L),$$

for $\psi_L = u_L, d_L$ etc. $\psi_R = u_R, d_R$ etc.

Thm A non-singular matrix can be diagonalized by a bi-unitary transform

i.e. $\exists$ U_R , U_L $U_R^\dagger U_R = U_L^\dagger U_L = 1$

Such that $U_L^\dagger \underline{m} U_R = \text{diag}(m_1, m_2\ m_3)$ $\equiv \underline{m}_d$.

$\underline{m} = \left[\delta^{AB} \frac{v}{\sqrt{2}} \right.$ - - $\cdot = u, d, e$.

Proof $\underline{m}\,\underline{m}^\dagger$ is Hermitian so $\exists$ U_L s.t. $(U_L^\dagger U_L = 1)$

$$U_L^\dagger \underline{m}\,\underline{m}^\dagger U_L = \underline{m}^2_{\text{diag}} \qquad m_i^2 > 0.$$

By a phase change $U_L \rightarrow U_L F$ $F = \text{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})$ we can take $\underline{m}_d$ to be real with $m_{d_i} > 0$

Put $H \equiv U_L \underline{m}_d U_L^\dagger$ $\Rightarrow H^\dagger = H$, and $V \equiv H^{-1}\underline{m}$

$\Rightarrow V^\dagger = \underline{m}^\dagger H^{-1}$, $VV^\dagger = H^{-1} m\, \underline{m}^\dagger H^{-1} = H^{-1} U_L\, \underline{m}^2_{\text{diag}} U_L^\dagger H^{-1}$

$= H^{-1} H^2 H^{-1} = 1.$

$H = H^\dagger, VV^\dagger = 1.$ So $\underline{m} = \underline{H} V$ - Polar decomposition. $U_L^\dagger H U = U_L^\dagger \underline{m} V^\dagger U_L = \underline{m}_d$; i' i.e $U_L^\dagger \underline{m}\, U_R = \underline{m}_{diag}$ $U_R \equiv V^\dagger U_L$.

So each mass matrix.

$\underline{m}_u$, $\underline{m}_d$, $\underline{m}_e$

Can be diagonalized by.

$u_L^A \to U_{uL}^{AB} u_L^B$, $u_R^A \to U_{uR}^{AB} u^B$

$d_L^A \to U_{dL}^{AB} d_L^B$ $d_R^A \to U_{dR}^{AB} d^B$

$e_L^A \to U_{eL}^{AB} e_L^B$ $e_R^A \to U_{eR}^{AB} e_R^B$.

$\nu_L^A \to U_{\nu L}^{AB} \nu_L^B$

$$\bar{\Psi}_L \underline{M}\, \Psi_R \to \bar{\Psi}_L U_L^\dagger M U_R \Psi_R = \bar{\Psi}_L \underline{m}_{diag} \Psi_R$$

* Note since $m^{AB} = f^{AB} v/\sqrt{2}$ diagonalises Yukawa Interaction also

However now gauge interactions are not diagonal ~~of~~ ($g W_\mu^r$ (or $W_\mu^\pm$)) ~~are~~ in the new basis. (charged current interactions).

$$\mathcal{L}_{Matter} \sim i \bar{Q}_L^A \gamma^\mu \left(-i \tfrac{g}{\sqrt{2}} \tfrac{\sigma^\pm}{2}\right) Q_L^A W_\mu^\pm \sim \frac{g}{\sqrt{2}} J_\mu^\pm W^{\mu\mp}.$$

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp i W_\mu^2\right)$$

Charged current

$\frac{\sigma^{\pm}}{2} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$$J_\mu^{\pm} = \bar{Q}_L^A \gamma_\mu \frac{\sigma^{\pm}}{2} Q_L = \bar{u}_L^A \gamma_\mu d_L^A$$

$$\rightarrow \cancel{\Phi_L} \; \bar{u}_L^A \gamma_\mu \left(U_{uL}^\dagger U_{dL} \right)^{AB} d_L^B$$

This may effectively be regarded as ~~a~~ rotation of the down quark basis

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix} \rightarrow U_{CKM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \equiv \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}$$

$$U_{CKM} \equiv U_{uL}^\dagger U_{dL} \quad \text{(CKM matrix)}$$

Note $U_{CKM} U_{CKM}^\dagger = 1$

Point is the basis in which the mass terms are diagonal is different from the basis in which the charge current is diagonal.

* Note neutral currents remain diagonal though ~~T~~ since they do not mix u or d quarks!

Note as long as there is no analog of d_R in the lepton sector i.e (no ν_R) can choose

$$U_{\nu L} = U_{eL} \Rightarrow U^{\dagger}_{eL} U_{eL} = 1$$

so no CKM for leptons.

[- Neutrino masses exist so
- need to modify this - later]

~~# of~~

Anyway this means lepton families are decoupled.

i.e. Seperate lepton # conservation

$$\pi^{\pm} \to l^{\pm} + \nu_l (\bar{\nu}_l)$$

~~ie~~ eg. $\mu \to e\gamma$ ~~$K^0 \to \mu e$~~ forbidden.

of parameters in CKM matrix.

3X3 Unitary matrix (9-real #)

(if U real then it is orthogonal 3×3 ⇒ 3-angles.)

- 3 real angles - 6 - phases.

However can remove 5 - out of 6 phases - by $u^A \to e^{i\varphi_{u^A}} u^A$ $d^A \to e^{i\varphi_{d^A}} d^A$

$A = 1, 2, 3$

, except that a common rotation on all six will leave U_{CKM} unaffected.

So, U_{CKM} has 3 - angles + 1 phase.

No phase (no CP) for 2-generations

1- phase for 3-generations

Generally if n-doublets : Complex matrix $2n^2$ real#

. # Unitarity conditions n^2 ⇒ Unitary matrix n^2 real #s.

angles = # of parameters in orthogonal matrix $= \frac{n(n-1)}{2}$.

phases which can be removed $= 2n-1$

physical phases $= n^2-(2n-1)-\frac{n(n-1)}{2} = \frac{(n-1)(n-2)}{2}$

$n=2$ no phases

$n=3$ 1- phase.