HW3 - Phys 7810-001

due 03/11/21

Problem 1

[40 pts]

Two Higgs doublet models: Suppose the Higgs doublet of the standard model is supplemented by a second complex doublet,ψ , transforming as $(1; \mathbf{2}; -1/2)$ under $SU_c(3) \times SU_L(2) \times U_Y(1)$.

a) [5 pts] If $\psi = \begin{pmatrix} \chi \\ \xi \end{pmatrix}$, what are the electric charges of the component fields χ, ξ?

b) [5 pts] Write out the covariant derivative $D_\mu \psi$ explicitly in terms of the gauge fields G^α_μ, W^a_μ and B_μ.

c) [10 pts] Assuming the potential must be a function of the invariants $a = \phi^\dagger \phi$, $b = \psi^\dagger \psi$, and $c = \phi^{\mathrm{T}} \epsilon \psi$, where ϕ is the usual Higgs doublet, what is the most general renormalizable form? How many independent real parameters does it contain? Need the parameters appearing in the potential be real? Is the combination $d = \phi^\dagger \psi$ $SU_L(2) \times U_Y(1)$ invariant?

d) [20 pts] Suppose the parameters of the potential are such that it is minimized when

$$\phi = \phi_{\min} = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}$$
$$\psi = \psi_{\min} = \begin{pmatrix} (u + iw)/\sqrt{2} \\ 0 \end{pmatrix},$$

with u, v, w all real. Do these values break the electromagnetic group $U_{em}(1)$ generated by the electric charge $Q = T_3 + Y$? Identify the terms in the

Lagrangian that are quadratic in the gauge fields and find their masses in terms of u, v, and w. Call the mass eigenstates $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_{\mu 1} \mp i W_{\mu 2})$, $Z = W_3 \cos\theta - B_\mu \sin\theta$, and $A_\mu = B_\mu \cos\theta + W_{3\mu} \sin\theta$. Express $\cos\theta$ in terms of the gauge couplings $SU_L(2) \times U_Y(1)$ gauge couplings. Is the standard model mass relation $M_W = M_Z \cos\theta$ also true for this model?

Problem 2

[40 pts]

Decay of the top quark: Consider the top quark, with a mass of $m_t = 173$ GeV. a) [15pts] Identify the only interaction term in the Lagrangian which is linear in the top quark. Can a single insertion of this interaction term cause the top quark to decay? What are the decay products? b) [15 pts] Write an expression for the matrix element for the dominant top quark decay process. Find a compact expression for the square of the matrix element, summing over final-state spin or helicity states and averaging over the initial top-quark helicity state. [10 pts] Compute the width of the top quark. Neglect the masses of any other fermions in comparison to the top-quark mass, but treat the masses of W and Z bosons as comparable to the top-quark mass. You should be able to find an analytic expression for the decay rate. Then, substitute in physical values and express the answer in GeV.

Problem 3

[20 pts] Consider the Abelian Higgs theory. After rewriting $A_\mu = Z_\mu$ and $(\mu, h) \to \frac{1}{\sqrt{2}}(\mu, h)$ you would have got after spontaneous symmetry breaking, the Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu h \partial^\mu h - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{\lambda}{4!}\mu^2 h^2 - \frac{\lambda}{4!}\mu h^3 \\ &- \frac{\lambda}{4 \times 4!} h^4 + \frac{1}{2} e^2 \mu^2 Z_\mu Z^\mu (1 + \frac{h}{\mu})^2 \end{aligned}$$

a) [10 pts] Write down the Feyman rules for this theory of Z-bosons and Higgs particles.

b) [10 pts] Calculate to lowest order the decay rate of Higgs particles into Z bosons. What inequality must λ, e satisfy in order for this process to occur.

(The differential decay rate in the rest frame of a decaying particle of mass M is given by $d\Gamma = \frac{1}{32\pi^2}|\mathcal{M}|^2 \frac{|\mathbf{p}|}{M^2} d\Omega$ where $\mathcal{M}$ is the invariant amplitude for decay into two particles and $\mathbf{p}$ is the three momentum of either of the decay particles.)